

Collaborative Relay Beamforming for Secrecy

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Abstract

In this paper, collaborative use of relays to form a beamforming system and provide physical-layer security is investigated. In particular, decode-and-forward (DF) and amplify-and-forward (AF) relay beamforming designs under total and individual relay power constraints are studied with the goal of maximizing the secrecy rates when perfect channel state information (CSI) is available. In the DF scheme, the total power constraint leads to a closed-form solution, and in this case, the optimal beamforming structure is identified in the low and high signal-to-noise ratio (SNR) regimes. The beamforming design under individual relay power constraints is formulated as an optimization problem which is shown to be easily solved using two different approaches, namely semidefinite programming and second-order cone programming. A simplified and suboptimal technique which reduces the computation complexity under individual power constraints is also presented. In the AF scheme, not having analytical solutions for the optimal beamforming design under both total and individual power constraints, an iterative algorithm is proposed to numerically obtain the optimal beamforming structure and maximize the secrecy rates. Finally, robust beamforming designs in the presence of imperfect CSI are investigated for DF-based relay beamforming, and optimization frameworks are provided.

Index Terms: amplify-and-forward relaying, decode-and-forward relaying, physical-layer security, relay beamforming, robust beamforming, second-order cone programming, secrecy rates, semidefinite programming.

I. INTRODUCTION

The broadcast nature of wireless transmissions allows for the signals to be received by all users within the communication range, making wireless communications vulnerable to eavesdropping. The problem of

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secure transmission in the presence of an eavesdropper was first studied from an information-theoretic perspective in [1] where Wyner considered a wiretap channel model. Wyner showed that secure communication is possible without sharing a secret key if the eavesdropper's channel is a degraded version of the main channel, and identified the rate-equivocation region and established the secrecy capacity of the degraded discrete memoryless wiretap channel. The secrecy capacity is defined as the maximum achievable rate from the transmitter to the legitimate receiver, which can be attained while keeping the eavesdropper completely ignorant of the transmitted messages. Later, Wyner's result was extended to the Gaussian channel in [3] and recently to fading channels in [4] and [5]. In addition to the single antenna case, secrecy in multi-antenna models is addressed in [6] and [7]. One particular result in [6] and [7] that is related to our study is that for the MISO secrecy channel, the optimal transmitting strategy is beamforming based on the generalized eigenvector of two matrices that depend on the channel coefficients. Regarding multiuser models, Liu *et al.* [8] presented inner and outer bounds on secrecy capacity regions for broadcast and interference channels. The secrecy capacity of the multi-antenna broadcast channel is obtained in [9].

Having multiple antennas at the transmitter and receiver has multitude of benefits in terms of increasing the performance, and provides the potential to improve the physical-layer security as well. Additionally, it is well known that even if they are equipped with single-antennas individually, users can cooperate to form a distributed multi-antenna system by performing relaying [13]–[15]. When channel side information (CSI) is exploited, relay nodes can collaboratively work similarly as in a MIMO system to build a virtual beam towards the receiver. Relay beamforming research has attracted much interest recently (see e.g., [16]–[20] and references therein). The optimal power allocation at the relays has been addressed in [17] and [18] when instantaneous CSI is known. In [19], the problem of distributed beamforming in a relay network is considered with the availability of second-order statistics of CSI. Most recently, Zheng *et al.* [20] have addressed the robust collaborative relay beamforming design by optimizing the weights of amplify-and-forward (AF) relays. They maximize the worst-case signal-to-noise ratio (SNR) assuming that CSI is imperfect but bounded. Transmit beamforming and receive beamforming strategies have been studied extensively for over a decade. A recent tutorial paper [12] provides an overview of advanced convex optimization approaches to both transmit, receive and network beamforming problems, and includes a

comprehensive list of references in this area.

Cooperative relaying under secrecy constraints was also recently studied in [21]–[23]. In [21], a decode-and-forward (DF) based cooperative protocol is considered, and a beamforming system is designed for secrecy capacity maximization or transmit power minimization. For amplify-and-forward (AF), suboptimal closed-form solutions that optimize bounds on secrecy capacity are proposed in [22]. However, in those studies, the analysis is conducted only under total relay power constraints and perfect CSI assumption. In this paper, we investigate the collaborative relay beamforming under secrecy constraints in the presence of both total and individual power constraints with the assumptions of perfect and imperfect channel knowledge.

More specifically, our contributions in this paper are as follows:

- 1) In DF, under total power constraints, we analytically determine the beamforming structure in the high- and low-SNR regimes.
- 2) In DF, under individual power constraints, not having analytical solutions available, we provide an optimization framework to obtain the optimal beamforming that maximizes the secrecy rate. We use the semidefinite relaxation (SDR) approach to approximate the problem as a convex semidefinite programming (SDP) problem which can be solved efficiently. We also provide an alternative method by formatting the original optimization problem as a convex second-order cone programming (SOCP) problem that can be efficiently solved by interior point methods. Also, we describe a simplified suboptimal beamformer design under individual power constraints.
- 3) In AF, we first obtain an expression for the achievable secrecy rate, and then we show that the optimal beamforming solution that maximizes the secrecy rate can be obtained by semidefinite programming with a two dimensional search for both total and individual power constraints.
- 4) Two robust beamforming design methods for DF relaying are described in the case of imperfect CSI.

The organization of the rest of the paper is as follows. In Section II, we describe the channel model and study the beamforming design for DF relaying under secrecy constraints. Beamforming for AF relaying is investigated in Section III. In Section IV, robust beamforming design in the case of imperfect CSI is studied. Numerical results for the performance of different beamforming schemes are provided in Section

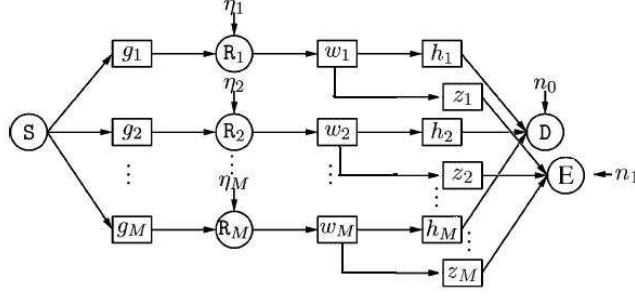


Fig. 1. Channel Model

V. Finally, we conclude in Section VI.

II. DECODE-AND-FORWARD RELAYING

We consider a communication channel with a source S , a destination D , an eavesdropper E , and M relays $\{R_m\}_{m=1}^M$ as depicted in Figure 1. In this model, the source S tries to transmit confidential messages to destination D with the help of the relays while keeping the eavesdropper E ignorant of the information. We assume that there is no direct link between S and D , and S and E . Hence, initially messages transmitted by the source are received only by the relays. Subsequently, relays work synchronously and multiply the signals with complex weights $\{w_m\}$ and produce a virtual beam point to the destination. We denote the channel coefficient between the source S and the m^{th} relay R_m as $g_m \in \mathbb{C}$, the channel coefficient between R_m and the destination D as $h_m \in \mathbb{C}$, and the channel coefficient between R_m and eavesdropper E as $z_m \in \mathbb{C}$.

It is obvious that our channel is a two-hop relay network. In the first hop, the source S transmits x_s to the relays with power $E[|x_s|^2] = P_s$. The received signal at R_m is given by

$$y_{r,m} = g_m x_s + \eta_m \quad (1)$$

where η_m is the background noise that has a complex, circularly symmetric Gaussian distribution with zero mean and variance of N_m .

In the second hop, we employ decode-and-forward transmission scheme. In this scheme, each relay first decodes the message x_s and normalizes it as $x'_s = x_s/\sqrt{P_s}$. Subsequently, the normalized message is multiplied by the weight factor w_m by the m^{th} relay to generate the transmitted signal $x_r = w_m x'_s$. The output power of the m^{th} relay R_m is given by

$$E[|x_r|^2] = E[|w_m x'_s|^2] = |w_m|^2. \quad (2)$$

The received signals at the destination D and eavesdropper E are the superpositions of the signals transmitted from the relays. These signals can be expressed, respectively, as

$$y_d = \sum_{m=1}^M h_m w_m x'_s + n_0 = \mathbf{h}^\dagger \mathbf{w} x'_s + n_0, \quad \text{and} \quad (3)$$

$$y_e = \sum_{m=1}^M z_m w_m x'_s + n_1 = \mathbf{z}^\dagger \mathbf{w} x'_s + n_1 \quad (4)$$

where n_0 and n_1 are the Gaussian background noise components at D and E , respectively, with zero mean and variance N_0 . Additionally, we have defined $\mathbf{h} = [h_1^*, \dots, h_M^*]^T$, $\mathbf{z} = [z_1^*, \dots, z_M^*]^T$, and $\mathbf{w} = [w_1, \dots, w_M]^T$ where superscript $*$ denotes conjugate operation, and $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose, respectively, of a matrix or vector. The metrics of interest are the received SNR levels at D and E , which are given, respectively, by

$$\Gamma_d = \frac{|\sum_{m=1}^M h_m w_m|^2}{N_0} \quad \text{and} \quad \Gamma_e = \frac{|\sum_{m=1}^M z_m w_m|^2}{N_0}. \quad (5)$$

It is well-known that given the channel coefficients, the secrecy rate R_s over the channel between the relays and destination is (see e.g., [3])

$$R_s = I(x'_s; y_d) - I(x'_s; y_e) \quad (6)$$

$$= \log(1 + \Gamma_d) - \log(1 + \Gamma_e) \quad (7)$$

$$= \log \left(\frac{N_0 + |\sum_{m=1}^M h_m w_m|^2}{N_0 + |\sum_{m=1}^M z_m w_m|^2} \right) \quad (8)$$

where $I(\cdot; \cdot)$ denotes the mutual information, and x'_s is Gaussian distributed with zero-mean and $E[|x'_s|^2] = 1$. Coding strategies that achieve the secrecy rates involve randomization at the encoder to introduce

uncertainty to the eavesdropper. Secrecy coding techniques are discussed in detail in [1] – [5]. Practical coding schemes for secure communications have been studied in [10] and [11] for certain special cases of the wiretap channel. It is important to note that we assume in the decode-and-forward scenario that the relays use the same secrecy codebook and transmit the same signal x'_s simultaneously. We further note that we throughout the text are interested in beamforming vectors that satisfy for given channel coefficients the inequality, $N_0 + |\sum_{m=1}^M h_m w_m|^2 > N_0 + |\sum_{m=1}^M z_m w_m|^2$. If there are no such beamforming vectors and the ratio inside the logarithm in (8) is less than 1, then the secrecy rate, by definition, is zero meaning that secure transmission cannot be established. The beamforming vectors which lead to zero secrecy capacity are not of interest.

In this section, we address the joint optimization of $\{w_m\}$ and hence identify the optimum collaborative relay beamforming (CRB) direction that maximizes the secrecy rate given in (8). Initially, we assume that the perfect knowledge of the channel coefficients is available. Later, in Section IV, we address the case in which the channel coefficients are only imperfectly known. We would like to also remark that the secrecy rate expression in (8) in a fading environment represents the instantaneous secrecy rate for given instantaneous values of the channel fading coefficients. Hence, in such a case, our formulation considers the optimization of $\{w_m\}$ in order to maximize the instantaneous secrecy rates.

A. Optimal Beamforming under Total Power Constraints

In this section, we consider a total relay power constraint in the following form: $\|\mathbf{w}\|^2 = \mathbf{w}^\dagger \mathbf{w} \leq P_T$. The optimization problem can now be formulated as follows:

$$\begin{aligned} R_s(\mathbf{h}, \mathbf{z}, P_T) &= \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \log \left(\frac{N_0 + |\sum_{m=1}^M h_m w_m|^2}{N_0 + |\sum_{m=1}^M z_m w_m|^2} \right) \\ &= \log \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{N_0 + |\sum_{m=1}^M h_m w_m|^2}{N_0 + |\sum_{m=1}^M z_m w_m|^2} \end{aligned} \quad (9)$$

$$= \log \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger (\frac{N_0}{P_T} \mathbf{I} + \mathbf{h} \mathbf{h}^\dagger) \mathbf{w}}{\mathbf{w}^\dagger (\frac{N_0}{P_T} \mathbf{I} + \mathbf{z} \mathbf{z}^\dagger) \mathbf{w}} \quad (10)$$

$$= \log \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \frac{\mathbf{w}^\dagger (N_0 \mathbf{I} + P_T \mathbf{h} \mathbf{h}^\dagger) \mathbf{w}}{\mathbf{w}^\dagger (N_0 \mathbf{I} + P_T \mathbf{z} \mathbf{z}^\dagger) \mathbf{w}} \quad (11)$$

$$= \log \lambda_{\max}(N_0 \mathbf{I} + P_T \mathbf{h} \mathbf{h}^\dagger, N_0 \mathbf{I} + P_T \mathbf{z} \mathbf{z}^\dagger) \quad (12)$$

where $\lambda_{\max}(\mathbf{A}, \mathbf{B})$ is the largest generalized eigenvalue of the matrix pair (\mathbf{A}, \mathbf{B}) ¹. Hence, the maximum secrecy rate in (12) is achieved by the optimal beamforming vector

$$\mathbf{w}_{opt} = \varsigma \mathbf{u} \quad (13)$$

where \mathbf{u} is the eigenvector that corresponds to $\lambda_{\max}(N_0\mathbf{I} + P_T\mathbf{h}\mathbf{h}^\dagger, N_0\mathbf{I} + P_T\mathbf{z}\mathbf{z}^\dagger)$ and ς is chosen to ensure $\mathbf{w}_{opt}^\dagger \mathbf{w}_{opt} = P_T$. Note that in the first-hop of the channel model, the maximum rate we can achieve is

$$R_1 = \min_{m=1,\dots,M} \log \left(1 + \frac{|g_m|^2 P_s}{N_m} \right). \quad (14)$$

Since we want all relays to successfully decode the signal transmitted from the source in the DF scenario, the rate expression in (14) is equal to the minimum of the rates required for reliable decoding at the relays. Hence, the first-hop rate is dictated by the worst channel among the channels between the source and the relays.

The overall secrecy rate is

$$R_{dof,s} = \min(R_1, R_s). \quad (15)$$

Above, we observe that having a severely weak source-relay channel can significantly degrade the performance. In these cases, other forwarding techniques (e.g., amplify-and-forward) can be preferred. Throughout the analysis of the DF scenario, we will not explicitly address these considerations and we will concentrate on the secure communication between the relays and the destination. Hence, we will have the implicit assumption that the source-relay links do not constitute a bottleneck for communication.

Next, we provide some remarks on the performance of collaborative relay beamforming in the high- and low-SNR regimes. Optimal beamforming under total power constraints is studied in detail in [21] and [22]. However, these studies have not identified the beamforming structure at low and high SNR levels. For simplicity, we assume in the following that the noise variances at the destination and eavesdropper are $N_0 = 1$.

¹For a Hermitian matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$ and positive definite matrix $\mathbf{B} \in \mathbb{C}^{n \times n}$, (λ, ψ) is referred to as a generalized eigenvalue – eigenvector pair of (\mathbf{A}, \mathbf{B}) if (λ, ψ) satisfy $\mathbf{A}\psi = \lambda\mathbf{B}\psi$ [28].

1) *High-SNR Regime:* In the high SNR scenario, where both $P_s, P_T \rightarrow \infty$, we can easily see that

$$\lim_{P_s \rightarrow \infty} (R_1 - \log P_s) = \min_{m=1, \dots, M} \log(|g_m|^2 / N_m). \quad (16)$$

From the Corollary 4 in Chapter 4 of [7], we can see that

$$\lim_{P_T \rightarrow \infty} (R_s - \log(P_T)) = \log(\max_{\tilde{\psi}} |\mathbf{h}^\dagger \tilde{\psi}|^2) \quad (17)$$

where $\tilde{\psi}$ is a unit vector on the null space of \mathbf{z}^\dagger . This result implies that choosing the beamforming vectors to lie in the null spaces of the eavesdropper's channel vector, i.e., having $|\sum_{m=1}^M z_m w_m|^2 = \mathbf{z}^\dagger \mathbf{w} = 0$, is asymptotically optimal in the high-SNR regime. In this case, the eavesdropper cannot receive any data from the relays, and secrecy is automatically guaranteed. No secrecy coding is needed at the relays. This asymptotic optimality can be seen from the following discussion. Assume that we impose the constraint $\mathbf{z}^\dagger \mathbf{w} = 0$. Now, the optimization problem (under the assumption $N_0 = 1$) becomes

$$\max_{\substack{\mathbf{w}^\dagger \mathbf{w} \leq P_T \\ \mathbf{z}^\dagger \mathbf{w} = 0}} \log \left(\frac{1 + \left| \sum_{m=1}^M h_m w_m \right|^2}{1 + \left| \sum_{m=1}^M z_m w_m \right|^2} \right) = \max_{\substack{\mathbf{w}^\dagger \mathbf{w} \leq P_T \\ \mathbf{z}^\dagger \mathbf{w} = 0}} \log \left(1 + \left| \sum_{m=1}^M h_m w_m \right|^2 \right) \quad (18)$$

$$= \max_{\substack{\hat{\mathbf{w}}^\dagger \hat{\mathbf{w}} \leq 1 \\ \mathbf{z}^\dagger \hat{\mathbf{w}} = 0}} \log \left(1 + \left| \sum_{m=1}^M h_m \hat{w}_m \sqrt{P_T} \right|^2 \right) \quad (19)$$

$$= \log(P_T) + \max_{\substack{\hat{\mathbf{w}}^\dagger \hat{\mathbf{w}} \leq 1 \\ \mathbf{z}^\dagger \hat{\mathbf{w}} = 0}} \log \left(\sqrt{\frac{1}{P_T}} + \left| \sum_{m=1}^M h_m \hat{w}_m \right|^2 \right) \quad (20)$$

$$\approx \log(P_T) + \log \left(\max_{\substack{\hat{\mathbf{w}}^\dagger \hat{\mathbf{w}} \leq 1 \\ \mathbf{z}^\dagger \hat{\mathbf{w}} = 0}} \left| \sum_{m=1}^M h_m \hat{w}_m \right|^2 \right) \quad (21)$$

$$= \log(P_T) + \log(\max_{\tilde{\psi}} |\mathbf{h}^\dagger \tilde{\psi}|^2) \quad (22)$$

such that $\mathbf{z}^\dagger \tilde{\psi} = 0$ and $\|\tilde{\psi}\|^2 = 1$. Above in (19), we have defined $\hat{\mathbf{w}} = \mathbf{w} / \sqrt{P_T}$ for which the constraint becomes $\hat{\mathbf{w}}^\dagger \hat{\mathbf{w}} \leq 1$. The approximation in (21) is due to the fact that $\frac{1}{\sqrt{P_T}}$ becomes negligible for large P_T . Hence, null space beamforming provides the same asymptotic performance as in (17) and is optimal in the high-SNR regime.

Furthermore, the optimal null space beamforming vector can be obtained explicitly. Due to the null space constraint, we can write $\mathbf{w} = \mathbf{H}_z^\perp \mathbf{v}$, where \mathbf{H}_z^\perp denotes the projection matrix onto the null space of \mathbf{z}^\dagger . Specifically, the columns of \mathbf{H}_z^\perp are orthonormal vectors that form the basis of the null space of \mathbf{z}^\dagger . In our case, \mathbf{H}_z^\perp is an $M \times (M - 1)$ matrix. The power constraint $\mathbf{w}^\dagger \mathbf{w} = \mathbf{v}^\dagger \mathbf{H}_z^{\perp\dagger} \mathbf{H}_z^\perp \mathbf{v} = \mathbf{v}^\dagger \mathbf{v} \leq P_T$. Then, the optimization problem can be recast as

$$\max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} \log \left(1 + \left| \sum_{m=1}^M h_m w_m \right|^2 \right) = \log \left(1 + \max_{\mathbf{w}^\dagger \mathbf{w} \leq P_T} (\mathbf{w}^\dagger \mathbf{h} \mathbf{h}^\dagger \mathbf{w}) \right) \quad (23)$$

$$= \log \left(1 + \max_{\mathbf{v}^\dagger \mathbf{v} \leq P_T} (\mathbf{v}^\dagger \mathbf{H}_z^{\perp\dagger} \mathbf{h} \mathbf{h}^\dagger \mathbf{H}_z^\perp \mathbf{v}) \right) \quad (24)$$

$$= \log \left(1 + P_T \lambda_{\max}(\mathbf{H}_z^{\perp\dagger} \mathbf{h} \mathbf{h}^\dagger \mathbf{H}_z^\perp) \right) \quad (25)$$

$$= \log \left(1 + P_T \mathbf{h}^\dagger \mathbf{H}_z^\perp \mathbf{H}_z^{\perp\dagger} \mathbf{h} \right). \quad (26)$$

Therefore, the optimum null space beamforming vector \mathbf{w} is

$$\mathbf{w}_{opt,n} = \mathbf{H}_z^\perp \mathbf{v} = \varsigma_1 \mathbf{H}_z^\perp \mathbf{H}_z^{\perp\dagger} \mathbf{h} \quad (27)$$

where ς_1 is a constant that is introduced to satisfy the power constraint.

2) *Low-SNR Regime:* In the low SNR regime, in which both $P_s, P_T \rightarrow 0$, we can see that

$$\lim_{P_s \rightarrow 0} \frac{R_1}{P_s} = \min_{m=1, \dots, M} \frac{|g_m|^2}{N_m}, \text{ and} \quad (28)$$

$$\lim_{P_s \rightarrow 0} \frac{R_s}{P_T} = \lambda_{\max}(\mathbf{h} \mathbf{h}^\dagger - \mathbf{z} \mathbf{z}^\dagger). \quad (29)$$

Thus, in the low SNR regime, the direction of the optimal beamforming vector approaches that of the eigenvector that corresponds to the largest eigenvalue of $\mathbf{h} \mathbf{h}^\dagger - \mathbf{z} \mathbf{z}^\dagger$. A similar result is shown in a multiple-antenna setting in [24].

B. Optimal Beamforming under Individual Power Constraints

In a multiuser network such as the relay system we study in this paper, it is practically more relevant to consider individual power constraints as wireless nodes generally operate under such limitations. Motivated

by this, we now impose $|w_m|^2 \leq p_m \forall m$ or equivalently $|\mathbf{w}|^2 \leq \mathbf{p}$ where $|\cdot|^2$ denotes the element-wise norm-square operation and \mathbf{p} is a column vector that contains the components $\{p_m\}$. In what follows, the problem of interest will be again be the maximization of the secrecy rate or equivalently the maximization of the term inside logarithm function of R_s (8) but now under individual power constraints:

$$\max_{|\mathbf{w}|^2 \leq \mathbf{p}} \frac{N_0 + |\sum_{m=1}^M h_m w_m|^2}{N_0 + |\sum_{m=1}^M z_m w_m|^2} \quad (30)$$

$$= \max_{|\mathbf{w}|^2 \leq \mathbf{p}} \frac{N_0 + \mathbf{w}^\dagger \mathbf{h} \mathbf{h}^\dagger \mathbf{w}}{N_0 + \mathbf{w}^\dagger \mathbf{z} \mathbf{z}^\dagger \mathbf{w}}. \quad (31)$$

1) Semidefinite Relaxation (SDR) Approach: We first consider a semidefinite programming method similar to that in [19]. Using the definition $\mathbf{X} \triangleq \mathbf{w} \mathbf{w}^\dagger$, we can rewrite the optimization problem in (31) as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{N_0 + \text{tr}(\mathbf{h} \mathbf{h}^\dagger \mathbf{X})}{N_0 + \text{tr}(\mathbf{z} \mathbf{z}^\dagger \mathbf{X})} \\ \text{s.t.} \quad & \text{diag}(\mathbf{X}) \leq \mathbf{p} \\ & \text{rank } \mathbf{X} = 1, \quad \text{and} \quad \mathbf{X} \succeq 0 \end{aligned} \quad (32)$$

or equivalently as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}(\mathbf{h} \mathbf{h}^\dagger - t \mathbf{z} \mathbf{z}^\dagger)) \geq N_0(t - 1), \\ & \text{diag}(\mathbf{X}) \leq \mathbf{p}, \\ & \text{rank } \mathbf{X} = 1, \quad \text{and} \quad \mathbf{X} \succeq 0 \end{aligned} \quad (33)$$

where $\text{tr}(\cdot)$ represents the trace of a matrix, $\text{diag}(\mathbf{X})$ denotes the vector whose components are the diagonal elements of \mathbf{X} , and $\mathbf{X} \succeq 0$ means that \mathbf{X} is a symmetric positive semi-definite matrix. The optimization problem in (33) is not convex and may not be easily solved. Let us now ignore the rank constraint in (33).

That is, using a semidefinite relaxation (SDR), we aim to solve the following optimization problem:

$$\begin{aligned}
& \max_{\mathbf{X}, t} \quad t \\
& s.t \quad \text{tr}(\mathbf{X}(\mathbf{h}\mathbf{h}^\dagger - t\mathbf{z}\mathbf{z}^\dagger)) \geq N_0(t - 1), \\
& \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0.
\end{aligned} \tag{34}$$

If the matrix \mathbf{X}_{opt} obtained by solving the optimization problem in (34) happens to be rank one, then its principal component will be the optimal solution to the original problem. Note that the optimization problem in (34) is quasiconvex. In fact, for any value of t , the feasible set in (34) is convex. Let t_{\max} be the maximum value of t obtained by solving the optimization problem (34). If, for any given t , the convex feasibility problem

$$\begin{aligned}
& \text{find } \mathbf{X} \\
& \text{such that } \text{tr}(\mathbf{X}(\mathbf{h}\mathbf{h}^\dagger - t\mathbf{z}\mathbf{z}^\dagger)) \geq N_0(t - 1), \\
& \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0
\end{aligned} \tag{35}$$

is feasible, then we have $t_{\max} \geq t$. Conversely, if the convex feasibility optimization problem (35) is not feasible, then we conclude $t_{\max} < t$. Therefore, we can check whether the optimal value t_{\max} of the quasiconvex optimization problem in (34) is smaller than or greater than a given value t by solving the convex feasibility problem (35). If the convex feasibility problem (35) is feasible then we know $t_{\max} \geq t$. If the convex feasibility problem (35) is infeasible, then we know that $t_{\max} < t$. Based on this observation, we can use a simple bisection algorithm to solve the quasiconvex optimization problem (34) by solving a convex feasibility problem (35) at each step. We assume that the problem is feasible, and start with an interval $[l, u]$ known to contain the optimal value t_{\max} . We then solve the convex feasibility problem at its midpoint $t = (l + u)/2$ to determine whether the optimal value is larger or smaller than t . We update the interval accordingly to obtain a new interval. That is, if t is feasible, then we set $l = t$, otherwise, we choose $u = t$ and solve the convex feasibility problem again. This procedure is repeated until the width of the interval is smaller than the given threshold. Note that the technique of using bisection search to solve the SDP feasibility problem is also given in [27]. Once the maximum feasible value for t_{\max} is obtained,

one can solve

$$\begin{aligned}
& \min_{\mathbf{X}} \quad \text{tr}(\mathbf{X}) \\
& \text{s.t.} \quad \text{tr}(\mathbf{X}(\mathbf{h}\mathbf{h}^\dagger - t_{\max}\mathbf{z}\mathbf{z}^\dagger)) \geq N_0(t_{\max} - 1), \\
& \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0
\end{aligned} \tag{36}$$

to get the solution \mathbf{X}_{opt} . (36) is a convex problem which can be solved efficiently using interior-point based methods.

To solve the convex feasibility problem, one can use the well-studied interior-point based methods as well. We use the well-developed interior point method based package SeDuMi [34], which produces a feasibility certificate if the problem is feasible, and its popular interface Yalmip [35]. In semidefinite relaxation, the solution may not be rank one in general. Interestingly, in our extensive simulation results, we have never encountered a case where the solution \mathbf{X}_{opt} to the SDP problem has a rank higher than one. In fact, there is always a rank one optimal solution for our problem as will be explained later. Therefore, we can obtain our optimal beamforming vector from the principal component of the optimal solution \mathbf{X}_{opt} .

2) *Second-order Cone Program (SOCP) Approach:* The reason that the SDR method is optimal for the above problem is that we can reformulate it as a second order cone problem [25] [26] by ignoring the phase in which we optimize \mathbf{w} directly rather than performing the optimization over $\mathbf{X} = \mathbf{w}\mathbf{w}^\dagger$. This provides us with another way of solving the optimization. The optimization problem (30) is equivalent to

$$\max_{\mathbf{w}, t} \quad t \tag{37}$$

$$\text{s.t.} \quad \frac{N_0 + |\mathbf{h}^\dagger \mathbf{w}|^2}{N_0 + |\mathbf{z}^\dagger \mathbf{w}|^2} \geq t \tag{38}$$

$$\text{and } |\mathbf{w}|^2 \leq \mathbf{p}.$$

Note that (38) can be written as

$$\frac{1}{t} |\mathbf{h}^\dagger \mathbf{w}|^2 \geq \left\| \left(\frac{\mathbf{z}^\dagger \mathbf{w}}{\sqrt{(1 - \frac{1}{t}) N_0}} \right) \right\|^2 = |\mathbf{z}^\dagger \mathbf{w}|^2 + \left(1 - \frac{1}{t}\right) N_0. \tag{39}$$

where the equality on the right hand side of (39) follows from the definition of the magnitude-square of a vector. The equivalence of (38) and (39) can easily be seen by rearranging the terms in (39). In the above formulation, we have implicitly assumed that $t \geq 1$. Note that this assumption does not lead to loss of generality as we are interested in cases in which $\frac{N_0 + |\mathbf{h}^\dagger \mathbf{w}|^2}{N_0 + |\mathbf{z}^\dagger \mathbf{w}|^2} > 1$. If this ratio is less than 1, the secrecy rate, as discussed before, is zero.

Observe that an arbitrary phase rotation can be added to the beamforming vector without affecting the constraint in (38). Thus, $\mathbf{h}^\dagger \mathbf{w}$ can be chosen to be real without loss of generality. We can take the square root of both sides of (39). The constraint becomes a second-order cone constraint, which is convex. The optimization problem now becomes

$$\begin{aligned} \max_{\mathbf{w}, t} \quad & t \\ \text{s.t.} \quad & \sqrt{\frac{1}{t}} \mathbf{h}^\dagger \mathbf{w} \geq \left\| \begin{pmatrix} \mathbf{z}^\dagger \mathbf{w} \\ \sqrt{\left(1 - \frac{1}{t}\right) N_0} \end{pmatrix} \right\| \text{ and } |\mathbf{w}|^2 \leq \mathbf{p}. \end{aligned} \quad (40)$$

As described in the SDR approach, the optimal solution of (40) can be obtained by repeatedly checking the feasibility and using a bisection search over t with the aid of interior point methods for second order cone program. Again, we use SeduMi together with Yalmip in our simulations. Once the maximum feasible value t_{\max} is obtained, we can then solve the following second order cone problem (SOCP) to obtain the optimal beamforming vector:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \|\mathbf{w}\|^2 \\ \text{s.t.} \quad & \sqrt{\frac{1}{t_{\max}}} \mathbf{h}^\dagger \mathbf{w} \geq \left\| \begin{pmatrix} \mathbf{z}^\dagger \mathbf{w} \\ \sqrt{\left(1 - \frac{1}{t_{\max}}\right) N_0} \end{pmatrix} \right\| \text{ and } |\mathbf{w}|^2 \leq \mathbf{p}. \end{aligned} \quad (41)$$

Thus, we can get the secrecy rate $R_{s,ind}$ for the second-hop relay beamforming system under individual power constraints employing the above two numerical optimization methods. Then, combined with the first-hop source relay link rate R_1 , secrecy rate of the decode and forward collaborative relay beamforming system becomes $R_{dof,ind} = \min(R_1, R_{s,ind})$.

3) *Simplified Suboptimal Design:* As shown above, the design of the beamformer under individual relay power constraints requires an iterative procedure in which, at each step, a convex feasibility problem is solved. We now propose a suboptimal beamforming vector that can be obtained without significant computational complexity.

We choose a simplified beamformer as $\mathbf{w}_{sim} = \theta \mathbf{w}_{opt}$ where \mathbf{w}_{opt} is given by (13) with $\|\mathbf{w}_{opt}\|^2 = P_T = \sum p_i$ where p_i is the individual power constraint for the i^{th} relay, and we choose

$$\theta = \frac{1}{|w_{opt,k}|/\sqrt{p_k}} \quad (42)$$

where $w_{opt,k}$ and p_k are the k th entries of \mathbf{w}_{opt} and \mathbf{p} respectively, and we choose k as

$$k = \arg \max_{1 \leq i \leq M} \frac{|w_{opt,i}|^2}{p_i} \quad (43)$$

Substituting this beamformer \mathbf{w}_{sim} into (8), we get the achievable suboptimal rate under individual power constraints.

III. AMPLIFY-AND-FORWARD RELAYING

Another common relaying scheme in practice is amplify-and-forward relaying. In this scenario, the received signal at the m^{th} relay R_m is directly multiplied by $l_m w_m$ without decoding, and forwarded to D . The relay output can be written as

$$x_{r,m} = w_m l_m (g_m x_s + \eta_m). \quad (44)$$

The scaling factor,

$$l_m = \frac{1}{\sqrt{|g_m|^2 P_s + N_m}}, \quad (45)$$

is used to ensure $E[|x_{r,m}|^2] = |w_m|^2$. The received signals at the destination D and eavesdropper E are the superposition of the messages sent by the relays. These received signals are expressed, respectively, as

$$y_d = \sum_{m=1}^M h_m w_m l_m (g_m x_s + \eta_m) + n_0, \text{ and} \quad (46)$$

$$y_e = \sum_{m=1}^M z_m w_m l_m (g_m x_s + \eta_m) + n_1. \quad (47)$$

Now, it is easy to compute the received SNR at D and E as

$$\Gamma_d = \frac{|\sum_{m=1}^M h_m g_m l_m w_m|^2 P_s}{\sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0}, \text{ and} \quad (48)$$

$$\Gamma_e = \frac{|\sum_{m=1}^M z_m g_m l_m w_m|^2 P_s}{\sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0}. \quad (49)$$

The secrecy rate is now given by

$$R_s = I(x_s; y_d) - I(x_s; y_e) \quad (50)$$

$$= \log(1 + \Gamma_d) - \log(1 + \Gamma_e) \quad (51)$$

$$= \log \left(\frac{|\sum_{m=1}^M h_m g_m l_m w_m|^2 P_s + \sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0}{|\sum_{m=1}^M z_m g_m l_m w_m|^2 P_s + \sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0} \right) \times \frac{\sum_{m=1}^M |z_m|^2 l_m^2 |w_m|^2 N_m + N_0}{\sum_{m=1}^M |h_m|^2 l_m^2 |w_m|^2 N_m + N_0}. \quad (52)$$

Again, we maximize this term by optimizing $\{w_m\}$ jointly with the aid of perfect CSI. It is obvious that we only have to maximize the term inside the logarithm function. Let us define

$$\mathbf{h}_g = [h_1^* g_1^* l_1, \dots, h_M^* g_M^* l_M]^T, \quad (53)$$

$$\mathbf{h}_z = [z_1^* g_1^* l_1, \dots, z_M^* g_M^* l_M]^T, \quad (54)$$

$$\mathbf{D}_h = \text{Diag}(|h_1|^2 l_1^2 N_1, \dots, |h_M|^2 l_M^2 N_M), \text{ and} \quad (55)$$

$$\mathbf{D}_z = \text{Diag}(|z_1|^2 l_1^2 N_1, \dots, |z_M|^2 l_M^2 N_M). \quad (56)$$

Then, the received SNR at the destination and eavesdropper can be reformulated, respectively, as

$$\Gamma_d = \frac{P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0} = \frac{P_s \text{tr}(\mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w} \mathbf{w}^\dagger)}{\text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger) + N_0}, \text{ and} \quad (57)$$

$$\Gamma_e = \frac{P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0} = \frac{P_s \text{tr}(\mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w} \mathbf{w}^\dagger)}{\text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger) + N_0}. \quad (58)$$

With these notations, we can write the objective function of the optimization problem as

$$\frac{1 + \Gamma_d}{1 + \Gamma_e} = \frac{1 + \frac{P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0}}{1 + \frac{P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0}} \quad (59)$$

$$= \frac{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_g \mathbf{h}_g^\dagger \mathbf{w}}{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0 + P_s \mathbf{w}^\dagger \mathbf{h}_z \mathbf{h}_z^\dagger \mathbf{w}} \times \frac{\mathbf{w}^\dagger \mathbf{D}_z \mathbf{w} + N_0}{\mathbf{w}^\dagger \mathbf{D}_h \mathbf{w} + N_0} \quad (60)$$

$$= \frac{N_0 + \text{tr}((\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger) \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}((\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger) \mathbf{w} \mathbf{w}^\dagger)} \times \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger)}. \quad (61)$$

If we denote $t_1 = \frac{N_0 + \text{tr}((\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger) \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}((\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger) \mathbf{w} \mathbf{w}^\dagger)}$, $t_2 = \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w} \mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w} \mathbf{w}^\dagger)}$, and use the similar SDR approach as described in the DF case, we can express the optimization problem as

$$\begin{aligned} & \max_{\mathbf{X}, t_1, t_2} \quad t_1 t_2 \\ & \text{s.t.} \quad \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\ & \quad \text{tr}(\mathbf{X}(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger))) \geq N_0(t_1 - 1) \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad (\text{and/or } \text{tr}(\mathbf{X}) \leq P_T) \quad \text{and} \quad \mathbf{X} \succeq 0. \end{aligned} \quad (62)$$

Notice that this formulation is applied to both total relay power constraint and individual relay power constraint which are represented by $\text{tr}(\mathbf{X}) \leq P_T$ and $\text{diag}(\mathbf{X}) \leq \mathbf{p}$, respectively. When there is only total power constraint, we can easily compute the maximum values of t_1 and t_2 separately since now we have Rayleigh quotient problems. These maximum values are

$$t_{1,u} = \lambda_{\max} \left(\mathbf{D}_h + \frac{N_0}{P_T} \mathbf{I} + P_s \mathbf{h}_g \mathbf{h}_g^\dagger, \mathbf{D}_z + \frac{N_0}{P_T} \mathbf{I} + P_s \mathbf{h}_z \mathbf{h}_z^\dagger \right), \quad (63)$$

$$t_{2,u} = \lambda_{\max} \left(\mathbf{D}_z + \frac{N_0}{P_T} \mathbf{I}, \mathbf{D}_h + \frac{N_0}{P_T} \mathbf{I} \right). \quad (64)$$

When there are individual power constraints imposed on the relays, we can use the bisection algorithm similarly as in the DF case to get the maximum values $t_{1,i,u}$ and $t_{2,i,u}$ ² for t_1 and t_2 by repeatedly solving the following two feasibility problems:

$$\begin{aligned}
& \text{find } \mathbf{X} \\
& \text{s.t. } \text{tr}(\mathbf{X}(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger))) \geq N_0(t_1 - 1) \\
& \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0,
\end{aligned} \tag{65}$$

and

$$\begin{aligned}
& \text{find } \mathbf{X} \\
& \text{s.t. } \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\
& \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0.
\end{aligned} \tag{66}$$

Note that for both total and individual power constraints, the maximum values of t_1 and t_2 are obtained separately above, and these values are in general attained by different $\mathbf{X} = \mathbf{w}\mathbf{w}^\dagger$. Now, the following strategy can be used to obtain achievable secrecy rates. For those \mathbf{X} values that correspond to $t_{1,i,u}$ and $t_{1,u}$ (i.e., the maximum t_1 values under individual and total power constraints, respectively), we can compute the corresponding $t_2 = \frac{N_0 + \text{tr}(\mathbf{D}_z \mathbf{w}\mathbf{w}^\dagger)}{N_0 + \text{tr}(\mathbf{D}_h \mathbf{w}\mathbf{w}^\dagger)}$ and denote them as $t_{2,i,l}$ and $t_{2,l}$ for individual and total power constraints, respectively. Then, $\log(t_{1,i,u} t_{2,i,l})$ and $\log(t_{1,u} t_{2,l})$ will serve as our amplify-and-forward achievable rates for individual and total power constraints, respectively. With the achievable rates, we propose the following algorithm to iteratively search over t_1 and t_2 to get the optimal $t_{1,o}$ and $t_{2,o}$ that maximize the product $t_1 t_2$

²Subscripts i in $t_{1,i,u}$ and $t_{2,i,u}$ are used to denote that these are the maximum values in the presence of individual power constraints.

by checking following feasibility problem.

$$\begin{aligned}
& \text{find} \quad \mathbf{X} \succeq 0 \\
& \text{s.t. } \text{tr}(\mathbf{X}(\mathbf{D}_z - t_2 \mathbf{D}_h)) \geq N_0(t_2 - 1) \\
& \quad \text{tr}(\mathbf{X}(\mathbf{D}_h + P_s \mathbf{h}_g \mathbf{h}_g^\dagger - t_1(\mathbf{D}_z + P_s \mathbf{h}_z \mathbf{h}_z^\dagger))) \geq N_0(t_1 - 1) \\
& \text{and } \text{tr}(\mathbf{X}) \leq P_T \quad \text{if there is total power constraint,} \\
& \text{or } \text{diag}(\mathbf{X}) \leq \mathbf{p} \quad \text{if there is individual power constraint.}
\end{aligned} \tag{67}$$

A. Proposed Algorithm

Define the resolution $\Delta t = \frac{t_{1,u}}{N}$ or $\Delta t = \frac{t_{1,i,u}}{N}$ for some large N for total and individual power constraints, respectively.

- 1) Initialize $t_{1,o} = t_{1,u}$, $t_{2,o} = t_{2,l}$ when total power constraint is imposed, and $t_{1,o} = t_{1,i,u}$, $t_{2,o} = t_{2,i,l}$ when individual power constraint is imposed. Initialize the iteration index $i = N$.
- 2) Set $t_1 = i\Delta t$. If $t_1 t_{2,u} < t_{1,o} t_{2,o}$ (total power constraint) or $t_1 t_{2,i,u} < t_{1,o} t_{2,o}$ (individual power constraint), then go to Step (3). Otherwise,
 - a) Let $t_2 = \frac{t_{1,o} t_{2,o}}{t_1}$. Check the feasibility problem (67). If it is infeasible, $i = i - 1$ go to step (2). If it is feasible, use the bisection algorithm in (67) with t_1 to get the maximum possible values of t_2 and denote this maximum as $t_{2,m}$. The initial interval in the above bisection algorithm can be chosen as $[\frac{t_{1,o} t_{2,o}}{t_1}, t_{2,u}]$ or $[\frac{t_{1,o} t_{2,o}}{t_1}, t_{2,i,u}]$ depending on the power constraints.
 - b) Update $t_{1,o} = t_1$, $t_{2,o} = t_{2,m}$, $i = i - 1$. Go back to step (2).

3) Solve the following problem to get the optimal \mathbf{X}

$$\begin{aligned}
& \min_{\mathbf{X}} \quad tr(\mathbf{X}) \\
& s.t \quad tr(\mathbf{X}(\mathbf{D}_z - t_{2,o}\mathbf{D}_h)) \geq N_0(t_{2,o} - 1) \\
& \quad \quad tr(\mathbf{X}(\mathbf{D}_h + P_s\mathbf{h}_g\mathbf{h}_g^\dagger - t_{1,o}(\mathbf{D}_z + P_s\mathbf{h}_z\mathbf{h}_z^\dagger))) \geq N_0(t_{1,o} - 1) \\
& \quad \quad \mathbf{X} \succeq 0 \text{ and} \\
& \quad \quad tr(\mathbf{X}) \leq P_T \quad \text{if there is total power constraint,} \\
& \quad \quad diag(\mathbf{X}) \leq \mathbf{p} \quad \text{if there is individual power constraint.}
\end{aligned} \tag{68}$$

B. Discussion of the Algorithm

Our algorithm is a two-dimensional search over all possible pairs (t_1, t_2) , which can produce the greatest product $t_1 t_2$, whose logarithm will be the global maximum value of the secrecy rate. In the following, we will illustrate how our algorithm works for individual power constraints. Similar discussion applies to the total power constraint case as well. The algorithm initiates with the achievable pair $(t_{1,i,u}, t_{2,i,l})$, in which $t_{1,i,u}$ is the maximum feasible value for t_1 . Thus, all t_1 values in our search lie in $[0, t_{1,i,u}]$. We chose the resolution parameter N to equally pick N points in this interval. We then use a brute force strategy to check each point iteratively starting from $t_{1,i,u}$ down to 0. In each iteration, the feasibility problem (67) is quasi-convex. Thus, we can use the bisection search over t_2 to get the greatest value of t_2 . Note that our initial bisection interval for t_2 is $[\frac{t_{1,o}t_{2,o}}{t_1}, t_{2,i,u}]$ where $t_{2,i,u}$ is the maximum feasible value for t_2 , and $\frac{t_{1,o}t_{2,o}}{t_1}$ is chosen so that the optimal t_2 we find at the end of the bisection search will produce a product $t_1 t_2$ that is greater than our currently saved optimal $t_{1,o}t_{2,o}$. With this approach, after each iteration, if a t_2 value is found, the new optimal $t_{1,o}t_{2,o}$ will be greater than the previous one. Note that our iteration's stop criterion is $t_1 t_{2,i,u} < t_{1,o}t_{2,o}$. This means that further decrease in the value of t_1 will not produce a product $t_1 t_2$ that is greater than our current $t_{1,o}t_{2,o}$. Thus, the value $t_{1,o}t_{2,o}$ at the end of this algorithm will be the global maximum since we have already checked all possible pairs t_1, t_2 that are candidates for the optimal value.

IV. ROBUST BEAMFORMING DESIGN

All of the beamforming methods discussed heretofore rely on the assumption that the exact knowledge of the channel state information is available for design. However, when the exact CSI is unavailable, the performance of these beamforming techniques may degrade severely. Motivated by this, the problem of robust beamforming design is addressed in [32] and [33]. The robust beamforming for MISO secrecy communications was studied in [29] where the duality between the cognitive radio MISO channel and secrecy MISO channel is exploited to transform the robust design of the transmission strategy over the secrecy channel into a robust cognitive radio beamforming design problem.

We additionally remark that, beside the assumption of perfect channel state information, our previous analysis is applicable only when the relays are fully synchronized at the symbol level. When the time synchronization between the relays is poor, the signal replicas passed through different relays will arrive to the destination node with different delays. This will result in inter-symbol-interference (ISI). To combat such ISI, the authors of [30] view an asynchronous flat-fading relay network as an artificial multipath channel (where each channel path corresponds to one particular relay), and use the orthogonal frequency division multiplexing (OFDM) scheme at the source and destination nodes to deal with this artificial multipath channel. In [31], a filter-and-forward protocol has been introduced for frequency selective relay networks, and several related network beamforming techniques have been developed. In these techniques, the relays deploy finite impulse response (FIR) filters to compensate for the effect of source-to-relay and relay-to-destination channels. Since the relay synchronization problem is out of the scope of this paper, we will mainly focus on combatting the effect of imperfect channel state information in the following discussion.

Systems robust against channel mismatches can be obtained by two approaches. In most of robust beamforming methods, the perturbation is modeled as a deterministic one with bounded norm which lead to a worst cast optimization. The other approach applied to the case in which the CSI error is unbounded is the statistical approach which provides the robustness in the form of confidence level measured by probability.

Let us consider the DF case. We define $\hat{\mathbf{H}} = \hat{\mathbf{h}}\hat{\mathbf{h}}^\dagger$ and $\hat{\mathbf{Z}} = \hat{\mathbf{z}}\hat{\mathbf{z}}^\dagger$ as the channel estimators, and $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$

and $\tilde{\mathbf{Z}} = \mathbf{Z} - \hat{\mathbf{Z}}$ as the estimation errors. First, consider the worst case optimization. In the worst case assumption, $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{Z}}$ are bounded in their Frobenius norm as $\|\tilde{\mathbf{H}}\| \leq \epsilon_H$, $\|\tilde{\mathbf{Z}}\| \leq \epsilon_Z$, where ϵ_H, ϵ_Z are assumed to be upper bounds of the channel uncertainty. Based on the result of [32], the robust counterpart of previously discussed SDR-based optimization problem can be written as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & \text{tr}(\mathbf{X}((\hat{\mathbf{H}} - \epsilon_H \mathbf{I}) - t(\hat{\mathbf{Z}} + \epsilon_Z \mathbf{I})) \geq N_0(t - 1), \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}, \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (69)$$

Note that the total power constraint $\text{tr}(\mathbf{X}) \leq P_T$ can be added into the formulation or substituted for the individual power constraint in (69). This problem can be solved the same way as discussed before.

However, the worst-case approach requires the norms to be bounded, which is usually not satisfied in practice. Also, this approach is too pessimistic since the probability of the worst-case may be extremely low. Hence, statistical approach is a good alternative in certain scenarios. In our case, we require the probability of the non-outage for secrecy transmission is greater than the predefined threshold ε by imposing

$$Pr \left(\frac{N_0 + \text{tr}((\hat{\mathbf{H}} + \tilde{\mathbf{H}})\mathbf{X})}{N_0 + \text{tr}((\hat{\mathbf{Z}} + \tilde{\mathbf{Z}})\mathbf{X})} \geq t \right) = Pr \left(\text{tr}(\mathbf{X}(\hat{\mathbf{H}} + \tilde{\mathbf{H}} - t(\hat{\mathbf{Z}} + \tilde{\mathbf{Z}})) \geq N_0(t - 1) \right) \geq \varepsilon. \quad (70)$$

Now, the optimization problem under imperfect CSI can be expressed as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & Pr \left(\text{tr}(\mathbf{X}(\hat{\mathbf{H}} + \tilde{\mathbf{H}} - t(\hat{\mathbf{Z}} + \tilde{\mathbf{Z}})) \geq N_0(t - 1) \right) \geq \varepsilon, \\ & \text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p} \text{ (or } \text{tr}(\mathbf{X}) \leq P_T), \quad \text{and } \mathbf{X} \succeq 0. \end{aligned} \quad (71)$$

If relays are under individual power constraints, we use $\text{diag}(\mathbf{X}) \leq \mathbf{p}$. Otherwise, for the case of total power constraint, we use $\text{tr}(\mathbf{X}) \leq P_T$. We can also impose both constraints in the optimization.

Note that the distribution of the components of the error matrices $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{Z}}$ depend on the channel estimation technique and distribution of the channel coefficients. In order to simplify the analysis and provide an analytically and numerically tractable approach, we assume that the components of the Hermitian

channel estimation error matrices $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{Z}}$ are independent, zero-mean, circularly symmetric, complex Gaussian random variables with variances $\sigma_{\tilde{H}}^2$ and $\sigma_{\tilde{Z}}^2$. Such an assumption is also used in [33]. Now, we can rearrange the probability in the constraint as

$$Pr\left(tr\left((\hat{\mathbf{H}} - t\hat{\mathbf{Z}} + \tilde{\mathbf{H}} - t\tilde{\mathbf{Z}})\mathbf{X}\right) \geq (t-1)N_0\right). \quad (72)$$

Let us define $y = tr\left((\hat{\mathbf{H}} - t\hat{\mathbf{Z}} + \tilde{\mathbf{H}} - t\tilde{\mathbf{Z}})\mathbf{X}\right)$. For given \mathbf{X} , $\hat{\mathbf{H}}$, and $\hat{\mathbf{Z}}$, we know from the results of [33] that y is a Gaussian distributed random variable with mean $\mu = tr\left((\hat{\mathbf{H}} - t\hat{\mathbf{Z}})\mathbf{X}\right)$ and variance $\sigma_y^2 = (\sigma_{\tilde{H}}^2 + t^2\sigma_{\tilde{Z}}^2)tr(\mathbf{X}\mathbf{X}^\dagger)$. Then, the non-outage probability can be written as

$$Pr(y \geq (t-1)N_0) = \int_{(t-1)N_0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(y-\mu)^2}{2\sigma_y^2}\right) \quad (73)$$

$$= \frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y}\right) \geq \varepsilon, \quad (74)$$

or equivalently as,

$$\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y} \leq \text{erf}^{-1}(-2\varepsilon + 1). \quad (75)$$

Note that ε should be close to one for good performance. Thus, both $-2\varepsilon + 1$ and $\frac{(t-1)N_0 - \mu}{\sqrt{2}\sigma_y}$ should be negative valued. Note further that we have $tr(\mathbf{X}\mathbf{X}^\dagger) = \|\mathbf{X}\|^2$, and hence $\sigma_y = \sqrt{\sigma_{\tilde{H}}^2 + t^2\sigma_{\tilde{Z}}^2}\|\mathbf{X}\|$. Then, this constraint can be written as

$$\|\mathbf{X}\| \leq \frac{(t-1)N_0 - \mu}{\sqrt{2(\sigma_{\tilde{H}}^2 + t^2\sigma_{\tilde{Z}}^2)}\text{erf}^{-1}(-2\varepsilon + 1)}. \quad (76)$$

As a result, the optimization problem becomes

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{s.t.} \quad & \|\mathbf{X}\| \leq \frac{(t-1)N_0 - \mu}{\sqrt{2(\sigma_{\tilde{H}}^2 + t^2\sigma_{\tilde{Z}}^2)}\text{erf}^{-1}(-2\varepsilon + 1)}, \end{aligned} \quad (77)$$

$$\text{and } \text{diag}(\mathbf{X}) \leq \mathbf{p}(\text{or } tr(\mathbf{X}) \leq P_T), \quad \text{and } \mathbf{X} \succeq 0.$$

Using the same bisection search, we can solve this optimization numerically.

V. NUMERICAL RESULTS

We assume that $\{g_m\}$, $\{h_m\}$, $\{z_m\}$ are complex, circularly symmetric Gaussian random variables with zero mean and variances σ_g^2 , σ_h^2 , and σ_z^2 respectively. We first provide numerical results for decode-and-forward beamforming schemes. In our numerical results, we focus on the performance of second-hop secrecy rate since the main emphasis of this paper is on the design of the beamforming system in the second-hop. Moreover, each figure is plotted for fixed realizations of the Gaussian channel coefficients. Hence, the secrecy rates in the plots are instantaneous secrecy rates.

In Figures 2 and 3, we plot the second-hop secrecy rate, which is the maximum secrecy rate that our collaborative relay beamforming system can support under both total and individual relay power constraints. For the case of individual relay power constraints, we assume that the relays have the same power budgets: $p_i = \frac{P_T}{M}$. Specifically, in Fig. 2, we have $\sigma_h = 3$, $\sigma_z = 1$, $N_0 = 1$ and $M = 5$. In this case, the legitimate user has a stronger channel. In Fig. 3, the only changes are $\sigma_h = 1$ and $\sigma_z = 2$, which imply that the eavesdropper has a stronger channel. Our CRB system can achieve secure transmission even when the eavesdropper has more favorable channel conditions. As can be seen from the figures, the highest secrecy rate is achieved, as expected, under a total transmit power constraint. On the other hand, we observe that only a relatively small rate loss is experienced under individual relay power constraints. Moreover, we note that our two different optimization approaches give nearly the same result. It also can be seen that under individual power constraint, the simple suboptimal method suffers a constant loss as compared to SDR or SOCP based optimal value.

In Fig. 4, we fix the relay total transmitting power as $P_T = 10dB$, and vary the number of collaborative relays. Other parameters are the same as those used in Fig. 3. We can see that increasing M , increases the secrecy rate under both total and individual power constraints. We also observe that in some cases, increasing M can degrade the performance when our simplified suboptimal beamformer is used.

In Fig. 5, we plot the secrecy rate for amplify-and-forward collaborative relay beamforming system for both individual and total power constraints. We also provide the result of suboptimal achievable secrecy rate for comparison. The fixed parameters are $\sigma_g = 10$, $\sigma_h = 2$, $\sigma_z = 2$, and $M = 10$. Since the AF secrecy rates depend on both the source and relay powers, the rate curves are plotted as a function of P_T/P_s . As

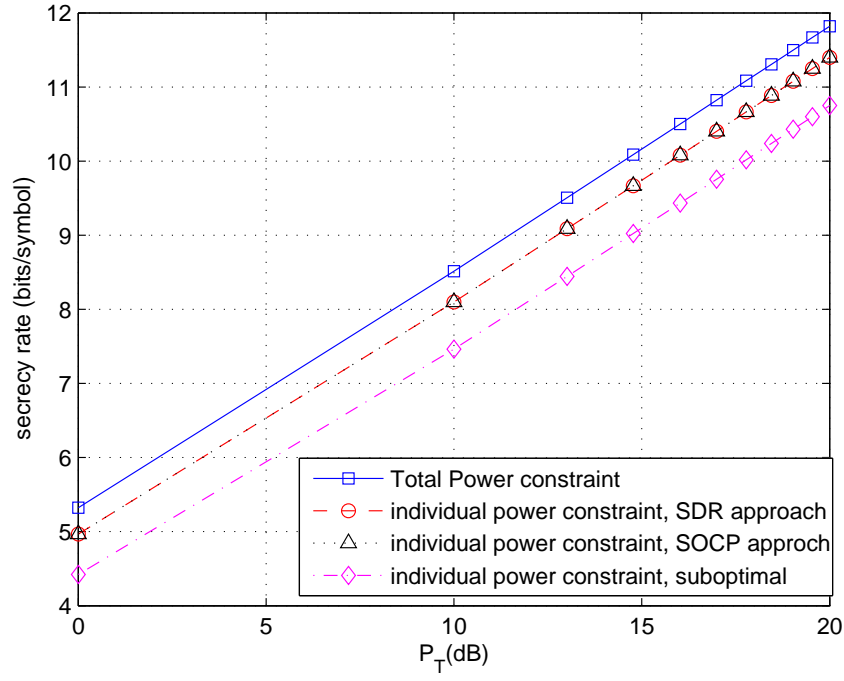


Fig. 2. DF Second-hop secrecy rate vs. the total relay transmit power P_T for different cases. Eavesdropper has a weaker channel.

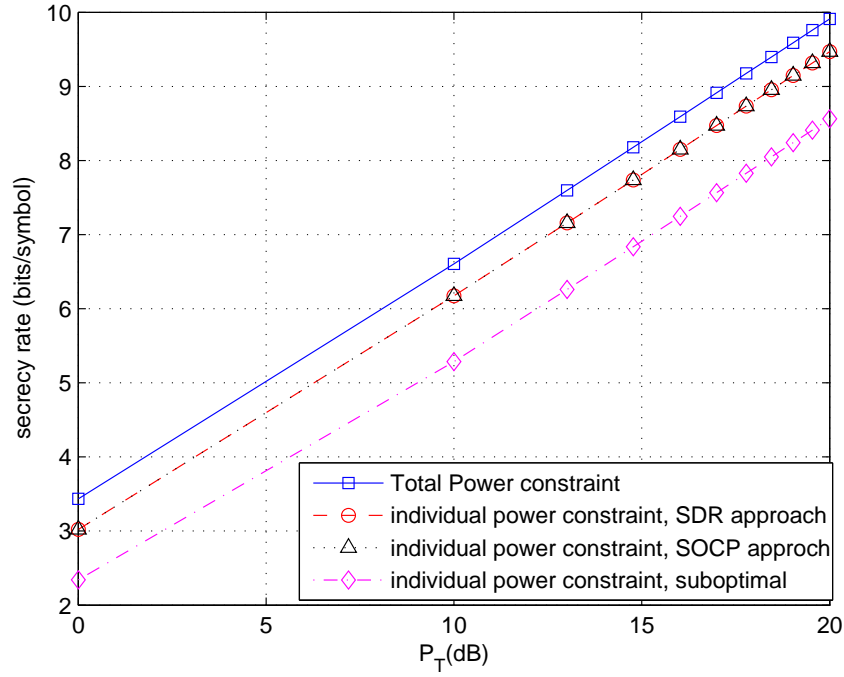


Fig. 3. DF Second-hop secrecy rate vs. the total relay transmit power P_T for different cases. Eavesdropper has a stronger channel.

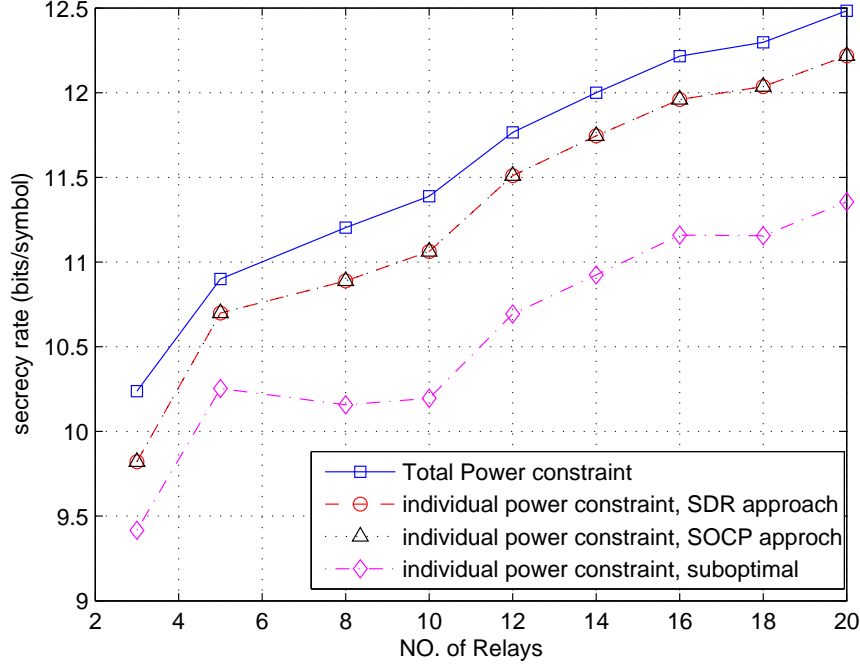


Fig. 4. DF second-hop secrecy rate vs. number of relays for different cases.

before, we assume that the relays have equal powers in the case in which individual power constraints are imposed, i.e., $p_i = P_T/M$. It is immediately seen from the figure that the achievable rates for both total and individual power constraints are very close to the corresponding optimal ones. Thus, the achievable beamforming scheme is a good alternative in the amplify-and-forward relaying case due to the fact that it has much less computational burden. Moreover, we interestingly observe that imposing individual relay power constraints leads to only small losses in the secrecy rates with respect to the case in which we have total relay power constraints.

In Fig. 6, we plot the maximum second hop secrecy rate of decode-and-forward that we can achieve for different power P_T and non-outage probability ε values. In this figure, we fix $M = 5$. $\hat{\mathbf{h}}$ and $\hat{\mathbf{z}}$ are randomly picked from Rayleigh fading with $\sigma_{\hat{h}} = 1$ and $\sigma_{\hat{z}} = 2$, and we assume that estimation errors are inversely proportional to P_T . More specifically, in our simulation, we have $\sigma_{\hat{H}}^2 = 0.1/P_T$ and $\sigma_{\hat{Z}}^2 = 0.2/P_T$. We also assume the relays are operating under equal individual power constraints, i.e., $p_i = \frac{P_T}{M}$. It is immediately observed in Fig. 6 that smaller rates are supported under higher non-outage probability requirements. In

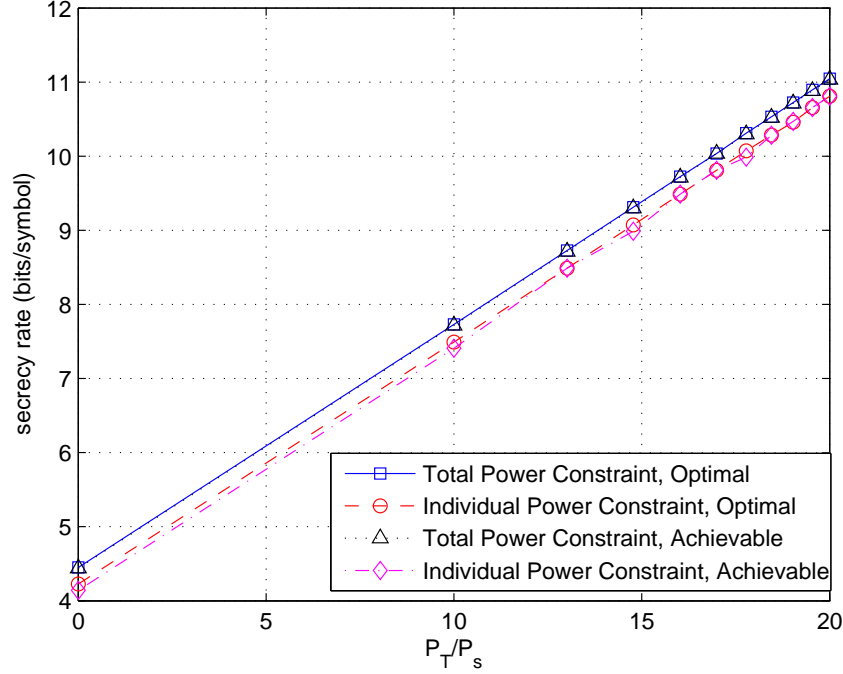


Fig. 5. AF secrecy rate vs. P_T/P_s . $\sigma_g = 10, \sigma_h = 2, \sigma_z = 2, M = 10$.

particular, this figure illustrates that our formulation and the proposed optimization framework can be used to determine how much secrecy rate can be supported at what percentage of the time. For instance, at $P_T = 20dB$, we see that approximately 7.4 bits/symbol secrecy rate can be attained 70 percent of the time (i.e., $\varepsilon = 0.7$) while supported secrecy rate drops to about 6.2 bits/symbol when $\varepsilon = 0.95$.

VI. CONCLUSION

In this paper, collaborative beamforming for both DF and AF relaying is studied under secrecy constraints. Optimal beamforming designs that maximize secrecy rates are provided under both total and individual relay power constraints. For DF with total power constraint, we have remarked that the optimal beamforming vector is the solution of a Rayleigh quotient problem. We have further identified the beamforming structure in the high- and low-SNR regimes. For DF with individual relay power constraints and AF with both total and individual relay power constraints, we have formulated the problem as a semidefinite programming problem and provided an optimization framework. We have also provided an alternative SOCP method to solve the DF relaying with individual power constraints. In addition, for DF relaying, we have described

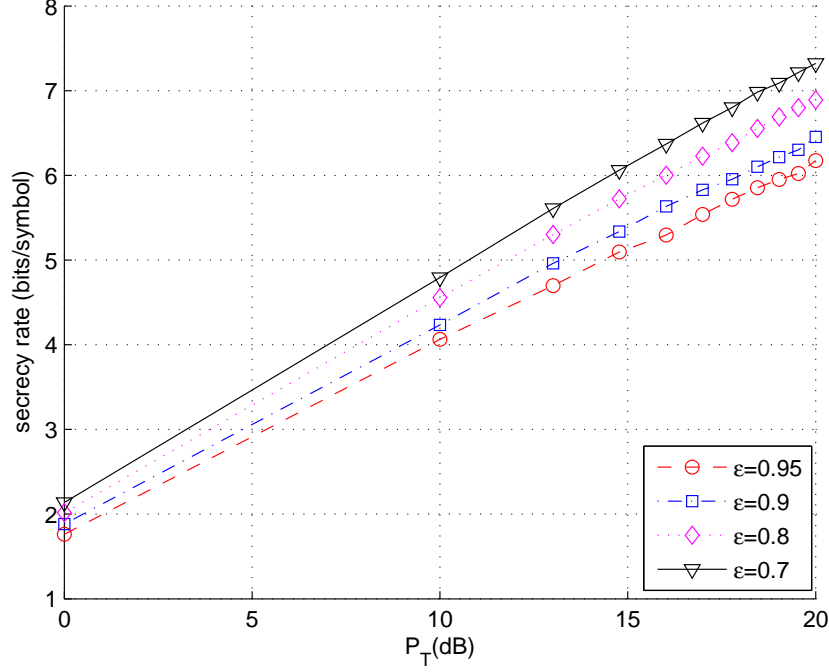


Fig. 6. DF second secrecy rate vs. P_T under different ϵ .

the worst-case robust beamforming design when CSI is imperfect but bounded, and the statistical robust beamforming design based upon minimum non-outage probability criterion. Finally, we have provided numerical results to illustrate the performance of beamforming techniques under different assumptions, e.g., DF and AF relaying, total and individual relay power constraints, perfect and imperfect channel information.

REFERENCES

- [1] A. Wyner "The wire-tap channel," *Bell. Syst. Tech. J.*, vol.54, no.8, pp.1355-1387, Jan 1975.
- [2] I. Csiszar and J. Korner "Broadcast channels with confidential messages," *IEEE Trans. Inform. Theory*, vol.IT-24, no.3, pp.339-348, May 1978.
- [3] S. K. Leung-Yan-Cheong and M. E. Hellman "The Gaussian wire-tap channel," *IEEE Trans. Inform. Theory*, vol.IT-24, no.4, pp.451-456, July 1978.
- [4] Y. Liang, H. V. Poor, and S. Shamai (Shitz), "Secure communication over fading channels," *IEEE Trans. Inform. Theory*, vol. 54, pp. 2470 - 2492, June 2008.
- [5] P. K. Gopala, L. Lai, and H. E. Gamal, "On the secrecy capacity of fading channels" *IEEE Trans. Inform. Theory*, vol.54, no.10, pp.4687-4698, Oct 2008.

- [6] S. Shafiee and S. Ulukus "Achievable rates in Gaussian MISO channels with secrecy constraint," *IEEE Intl Symp. on Inform. Theory*, Nice, France, June 2007.
- [7] A. Khisti, "Algorithms and architectures for multiuser, multiterminal, and multilayer information-theoretic security " Doctoral Thesis, MIT 2008.
- [8] R. Liu, I. Maric, P. Spasojevic and R. D. Yates, "Discrete memoryless interference and broadcast channels with confidential messages: Secrecy capacity regions," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2493-2507, Jun. 2008.
- [9] R. Liu and H. V. Poor, "Secrecy capacity region of a multi-antenna Gaussian broadcast channel with confidential messages," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1235-1249, Mar. 2009.
- [10] H. Mahdaviar and A. Vardy, "Achieving the secrecy capacity of wiretap channels using polar codes," submitted to 2010 IEEE International Symposium on Information Theory. Available online at <http://arxiv.org/abs/1001.0210>.
- [11] A. Thangaraj, S. Dihidar, A.R. Calderbank, S.W. McLaughlin and J. Merolla, "Applications of LDPC codes to the wiretap channel," *IEEE Trans. Inform. Theory*, vol. 53, no. 8, pp. 2933-2945, August 2007.
- [12] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming: from receive to transmit and network designs," to appear in *IEEE Signal Proc. Magazine*, 2010. Available online at http://www.kth.se/ees/forskning/publikationer/modules/publications_polopoly/reports/2010/IR-EE-SB_2010_007.pdf
- [13] J. N. Laneman, D. N. C. Tse, G. W. Wornel "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062-3080. Dec. 2004
- [14] R. U. Nabar, H. Bolcskei, F. W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Select. Areas Commun* vol. 22, NO. 6 pp. 1099-1109. Aug. 2004
- [15] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037-3063, Sept. 2005
- [16] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Honolulu, Hawaii, Apr. 15-20, 2007.
- [17] G. Zheng, K. Wong, A. Paulraj, and B. Ottersten, "Collaborative-relay beamforming with perfect CSI: Optimum and distributed implementation," *IEEE Signal Process Letters*, vol. 16, no. 4, Apr. 2009
- [18] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Inf. Theory*, vol. 55, pp. 2499-2517, June 2009
- [19] V. Nussab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second order statistics of the channel state information," *IEEE Trans. on Signal Proc.*, Vol. 56, No 9, pp. 4306-4316, Sept. 2008.
- [20] G. Zheng, K. K. Wong, A. Paulraj, and B. Ottersten, "Robust collaborative-relay beamforming," *IEEE Trans. on Signal Proc.*, vol. 57, no. 8, Aug. 2009
- [21] L. Dong, Z. Han, A. Petropulu and H. V. Poor, "Secure wireless communications via cooperation," *Proc. 46th Annual Allerton Conf. Commun., Control, and Computing*, Monticello, IL, Sept. 2008.
- [22] L. Dong, Z. Han, A. Petropulu, and H. V. Poor, "Amplify-and-forward based cooperation for secure wireless communications," *Proc. IEEE Intl Conf. Acoust. Speech Signal Proc.*, Taipei, Taiwan, Apr. 2009.

- [23] V. Aggarwal, L. Sankar, A. R. Calderbank, and H. V. Poor, "Secrecy capacity of a class of orthogonal relay eavesdropper channels," Available: <http://arxiv.org/abs/0812.2275>
- [24] M. C. Gursoy, "Secure communication in the low-SNR regime: A characterization of the energy-secrecy tradeoff," Proc. of the IEEE International Symposium on Information Theory (ISIT), Seoul, Korea, July 2009.
- [25] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. on Signal Proc.*, vol. 54, no. 3, Jan. 2006
- [26] H. Chen, A. B. Gershman, S. Shahbazpanahi, "Distributed peer-to-peer beamforming for multiuser relay networks," *Proc. IEEE Intl Conf. Acoust. Speech Signal Proc.*, Taipei, Taiwan, Apr. 2009.
- [27] L. Zhang, R. Zhang, Y.-C. Liang, Y. Xin, and S. Cui, "On the relationship between the multi-antenna secrecy communications and cognitive radio communications," *Proc. 47th Annual Allerton Conf. Commun., Control, and Computing*, Monticello, IL, Sept. 2009.
- [28] G. Golub and C. F. Van Loan, *Matrix Computations* (3rd ed), Johns Hopkins University Press, 1996
- [29] L. Zhang, Y.-C. Liang, Y. Pei, and R. Zhang, "Robust beamforming design: from cognitive radio MISO channels to secrecy MISO channels," *IEEE Globecom*, 2009.
- [30] A. Abdelkader, S. Shahbazpanahi, and A. B. Gershman, "Joint subcarrier power loading and distributed beamforming in OFDM-based asynchronous relay networks," *IEEE CAMSAP*, Aruba, Dec. 2009,
- [31] H. Chen, A. B. Gershman, and S. Shahbazpanahi, "Filter-and-forward distributed beamforming in relay networks with frequency selective fading," *IEEE Trans. on Signal Proc.*, vol. 58, no. 3, March. 2010
- [32] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," in *Proc. 37th Annual Allerton Conference on Communication, Control and Computing* Sep. 1999
- [33] B. K. Chalise and A. Czylik, "Robust uplink beamforming based upon minimum outage probability criterion," *IEEE Globecom*, 2004.
- [34] J. Sturm, "Using SeDuMi 1.02: A MATLAB toolbox for optimization over symmetric cones," *Opt. Methods and Software*, vol. 11-12, pp. 625-653, 1999, Special issue on Interior Point Methods (CD supplement with software).
- [35] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," *Proc. the CACSD Conf.*, Taipei, Taiwan, 2004. [Online]. Available: <http://control.ee.ethz.ch/~joloef/yalmip.php>.
- [36] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.